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Left-Right Asymmetry in $\bar{p}p \to \pi^-\pi^+$

George Bathas[a] and W.M. Kloet[a,b]

a. Department of Physics and Astronomy, Rutgers University
P.O. Box 849 Piscataway NJ 08855-0849 USA

b. Division de Physique Théorique, Institut de Physique Nucléaire,
F-91406, Orsay Cedex, France

Abstract

Results for dσ/dΩ and A_N in the reaction $\bar{p}p \to \pi^-\pi^+$ are predicted by a simple quark model. They are compared to recent experimental data from LEAR, as well as to previous predictions from nucleon-exchange models. At low energy the quark model does better than the nucleon-exchange models, but the overall comparison to experiment remains poor. In particular, the double-dip structure of the experimental A_N data is only partly represented. This shortcoming of the simple quark model is traced back to a too small J=2 amplitude. This has interesting implications for the range of this specific annihilation process.

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1. Introduction

Large left-right asymmetries in the reaction $\bar{p}p \rightarrow \pi^-\pi^+$ are observed [1-5] when the target proton is polarized perpendicularly to the scattering plane. For example, when the spin of the target proton is up, the π^- particles prefer to emerge to the right of the $\bar{p}$ beam-direction at low initial antiproton momenta (e.g. $\mathrm{p_{LAB}}$ = 500 MeV/c, or $\mathrm{T_{LAB}}$ = 124 MeV), while at high momenta (e.g. $\mathrm{p_{LAB}}$ = 1500 MeV/c, or $\mathrm{T_{LAB}}$ = 832 MeV) the π^- particles prefer to emerge to the left.

The reaction $\bar{p}p \rightarrow \pi^-\pi^+$ is one of the simplest $\bar{p}p$ annihilation channels because it depends on only two helicity amplitudes. In terms of partial waves it depends only on the spin-triplet coupled waves, while isospin takes the values I=0 for even J and I=1 for odd J. Therefore the existence of accurate data, in particular for the analyzing power A_N (also called A_y) is a challenge to microscopic models (like the quark model) for the $\bar{p}p$ annihilation into that particular channel.

For a detailed review of $\bar{p}p$ annihilation in general we refer to ref.[6]. Previous work that explicitly compares theoretical predictions for the above reaction $\bar{p}p \rightarrow \pi^-\pi^+$ with the available dσ/dΩ and A_N data is based on an annihilation mechanism driven by nucleon- and delta-exchange [5,7,8]. Reasonable agreement with the experimental structure of A_N is obtained at antiproton momenta in the range of 650-1100 MeV/c ($\mathrm{T_{LAB}}$ = 203-575 MeV). The agreement improves towards the higher end of this momentum range. However, at lower momenta ($\mathrm{p_{LAB}}$ = 467-585 MeV/c, $\mathrm{T_{LAB}}$ = 110-167 MeV), so far no model has predicted an asymmetry of overall negative sign at all angles, as is observed experimentally (see ref.[4,5]). Some reservation exists about nucleon-exchange models. Certainly, a legitimate concern of models based on nucleon-exchange is the distant extrapolation of the πNN form factor, from on-shell nucleons, into a region where one of the nucleons has become spacelike. Typically in this extrapolation an effective cutoff-mass of 1.5-1.7 GeV has been used.

It is therefore of interest to investigate an alternative approach to a microscopic annihilation mechanism, which is based on the constituent quark model. In a simple quark model one does not specify which object is being exchanged between the anti-proton and the proton, but rather how the three quarks and three anti-quarks in the initial state can be rearranged and partly annihilated in order to form two $q\bar{q}$ pairs, representing the final two pions.

In this paper we study the predictions at low energy for the differential cross section dσ/dΩ and the analyzing power A_N if the annihilation mechanism is described by a simple quark model. We

compare the results of the quark model with two other models where the annihilation mechanism is given by nucleon-exchange. From comparing with the experimental data from refs.[4,5], we are able to draw some conclusions about the specific annihilation of $\bar{p}p$ into $\pi^-\pi^+$.

In a more general context, one may also foster the hope that because of the short range nature of the annihilation process the reaction $\bar{p}p \longrightarrow \pi^-\pi^+$ could be a testing ground for possible differences between a description in terms of quarks and a description in terms of mesons, nucleons, and isobars.

2. Annihilation mechanism

To describe the annihilation of anti-proton proton into two pions we employ here the generalized $^3P_0$ and $^3S_1$ mechanism of ref.[9] . The proton (and antiproton) wave function is described as a Gaussian

$$\Psi_N(\vec{r}_1, \vec{r}_2, \vec{r}_3) \;=\; N_N \,\exp\,[\; -\frac{\alpha}{2}\,\Sigma\,(\vec{r}_{\rm i} \;-\; \vec{r}_{\rm N})^2 \;]\; X_N \;(\text{spin},\;\text{isospin},\;\text{color}), \tag{1}$$

where $\vec{r}_{\rm i}$ are the quark coordinates and $\vec{r}_{\rm N}$ is the nucleon coordinate. If we take the pion to be an S-wave meson, its wavefunction is given by

$$\Phi_M\;(\vec{r}_1, \vec{r}_4) \;=\; N_M\,\exp\,[\; -\frac{\beta}{2}\,\Sigma\,(\vec{r}_{\rm i} \;-\; \vec{r}_{\rm M})^2\; X_M \;(\text{spin},\;\text{isospin},\;\text{color}), \tag{2}$$

where $\vec{r}_1$ and $\vec{r}_4$ are respectively the quark and antiquark coordinates, and $\vec{r}_{\rm M}$ is the coordinate of the meson. Typical parameter values are $\alpha = 2.8$ fm^{-2} and $\beta = 3.23$ fm^{-2}, giving a nucleon radius of 0.60 fm and a meson radius of 0.48 fm.

The transition potential for this process, predicted by the simple quark model, was worked out in ref.[9]. It turns out to be a non-local potential, dependent on the relative $\bar{p}p$ coordinate $\vec{r}$, the relative $\pi\pi$ coordinate $\vec{r}\,'$, and the Pauli spin operator $\vec{\sigma}$. If one allows for two mechanisms, the $^3P_0$ and $^3S_1$ mechanisms, to be responsible for the annihilation of one of the $q\bar{q}$ pairs, the potential can be written as

$$V_{\rm ann} \;=\; V(^3P_0) \;+\; \lambda\, V(^3S_1). \tag{3}$$

We repeat here the form of V($^3P_0$) derived in ref.[9]:

$$
\begin{aligned}
V(^3P_0)(\vec{r}\,',\vec{r}) &\sim \\
\{A_V\ i\vec{\sigma}.\vec{r}\,'\sinh(C\vec{r}\,'.\vec{r}) &+ B_V\ i\vec{\sigma}.\vec{r}\cosh(C\vec{r}\,'.\vec{r})\}\exp(A\ r'^2\ +\ B\ r^2),
\end{aligned} \tag{4}
$$

$$
\text{where } A_V = \frac{\alpha(\alpha\ +\ \beta)}{2(4\alpha\ +\ 3\beta)}, \tag{5}
$$

$$
B_V = \frac{3(5\alpha^2\ +\ 8\alpha\beta\ +\ 3\beta^2)}{2(4\alpha\ +\ 3\beta)}, \tag{6}
$$

$$
A = -\frac{\alpha(5\alpha\ +\ 4\beta)}{2(4\alpha\ +\ 3\beta)}, \tag{7}
$$

$$
B = -\frac{3(7\alpha^2\ +\ 18\alpha\beta\ +\ 9\beta^2)}{8(4\alpha\ +\ 3\beta)}, \tag{8}
$$

$$
C = -\frac{3\alpha(\alpha\ +\ \beta)}{2(4\alpha\ +\ 3\beta)}. \tag{9}
$$

The expression for V($^3S_1$) consists of a longitudinal part and a transversal part. The longitudinal part has the same general form as eq.(4), the transversal part is obtained from eq.(4) by interchanging sinh and cosh. Both expressions, with the appropriate coefficients are given in ref.[9]. The parameter λ in eq.(3) gives the relative strength of the two mechanisms. The range and the nature of the non-localities of $V_{\rm ann}$ is a function of the size-parameters α and β of the proton and the pion respectively, mentioned in eqs.(1,2). The annihilation potential $V_{\rm ann}$ of eq.(3) contributes to all total angular momenta J of the $\bar{p}p$ system.

3. Observables and basic amplitudes

As initial state we choose the $\bar{p}p$ wavefunction as given by the 1982 Paris $\bar{p}p$ model [10] including the Coulomb interaction. As final state we choose non-interacting pions. Later, we will discuss the implications of different cases of initial state interactions and final state interactions, which may be substantial [11], in a different paper.

The observables, which have been measured in this reaction, are the differential cross section $d\sigma/d\Omega$ and the analyzing power A_N. Both observables are expressed in the two helicity amplitudes $F_{++}(\theta)$ and $F_{+-}(\theta)$ which fully describe the reaction $\bar{p}p \rightarrow \pi^-\pi^+$. The angle θ is the CM angle between the outgoing π^- and the incoming anti-proton. The observables considered are

$$d\sigma/d\Omega \;=\; (\,|F_{++}|^2 \;+\; |F_{+-}|^2)/2, \tag{10}$$

$$A_N \; d\sigma/d\Omega \;=\; Im\;(F_{++}\;F^*_{+-}). \tag{11}$$

For completeness and later reference, we note the relation of the two helicity amplitudes to the angular momentum amplitudes $f^{\rm J}_\ell$ and the Legendre polynomials $P_J(\cos\theta)$ and their derivatives $P'_J(\cos\theta)$. The indices J and ℓ are respectively the total angular momentum, and orbital angular momentum of the $\bar{p}p$ system. The angular momentum of the $\pi^-\pi^+$ system is $\ell_{\pi\pi}$ = J. We have [12]

$$F_{++}(\theta) \;=\; \frac{1}{p}\sum_{\rm J}\;\sqrt{(2J+1)/2}\;\;(\;\sqrt{J}\;f^J_{J-1}\;-\;\sqrt{(J+1)}\;f^J_{J+1})\;P_J(\cos\theta), \tag{12}$$

and

$$F_{+-}(\theta) \;=\; \frac{1}{p}\sum_{\rm J}\;\sqrt{(2J+1)/2}\;\;(\sqrt{1/J}\;f^J_{J-1}\;+\;\sqrt{1/(J+1)}\;f^J_{J+1})\;P'_J(\cos\theta). \tag{13}$$

The angular momentum amplitudes f^J_ℓ are basically two-dimensional integrals in r' and r of the type

$$I_{iJ\ell} \;=\; \int\!\!\int r'^2 dr' r^2 dr \Phi^{\pi\pi}_J(r')\;V_{iJ\ell}(r',r)\;\Psi^{\bar{p}p}_{J\ell}(r). \tag{14}$$

Here $V_{iJ\ell}(r',r)$ originates from the angular momentum decomposition of the transition potential $V_{\rm ann}(\vec{r}\,',\vec{r},\vec{\sigma})$ and takes on two possible forms

$$V_{1J\ell}(r',r) = r\;\exp\;[A\;r'^2\;+\;B\;r^2]\;\;j^{\rm mod}_J(Cr'r), \tag{15}$$

and

$$V_{2J\ell}(r', r) \; = \; r' \; \exp \; [A \; r'^2 \; + \; B \; r^2] \;\; j_\ell^{\text{mod}}(Cr'r). \tag{16}$$

The symbol j_ℓ^{mod} in eqs.(15,16) stands for the modified spherical Bessel function of the first kind of order ℓ. In eq. (14) above $\Phi_J^{\pi\pi}(r')$ and $\Psi_{J\ell}^{\bar{p}p}$ (r) are the final $\pi\pi$ and initial $\bar{p}p$ wavefunctions respectively.

As expected the integral of the above eq.(14) probes the $\bar{p}p$ wavefunction and $\pi\pi$ wavefunction at rather short distances since the values of A, B, and C are typically A = -1.80 fm^{-2}, B = -5.59 fm^{-2}, and C = -1.21 fm^{-2}. On the other hand one needs to realize that, for example, the wave function $\Psi_{J\ell}^{\bar{p}p}$ (r) vanishes at short distances r. This is caused by the total $\bar{p}p$ annihilation predominantly in channels different from $\pi\pi$, which, in case of the Paris model [10] is described by a phenomenological annihilation potential. This of course means to some extent a double counting of the specific annihilation into $\pi\pi$. But since the annihilation into $\pi\pi$ only accounts for about a percent of the total annihilation, double counting due to use of the Paris model for the initial $\bar{p}p$ interaction can be ignored. We can safely use this method to study the microscopic behaviour of the specific annihilation $\bar{p}p \to \pi^-\pi^+$. From the above it is clear that integrals of the type of eq.(14) are sensitive to the model used for $\bar{p}p$ initial state interaction, and at the same time sensitive to the model for $\pi\pi$ final state interaction. We return to this aspect in a separate paper.

4. Results

If we take for $\Psi_{J\ell}^{\bar{p}p}$ (r) the Paris model of 1982 [10] and for $\Phi_J^{\pi\pi}$ (r') simply plane waves, (no $\pi\pi$ interaction as they go out) , we obtain the result at p_{LAB} = 497 MeV/c (T_{LAB} = 123 MeV) for differential cross section dσ/dΩ and analyzing power A_N, shown in fig.1 for the case λ = - 0.5. The parameter λ which gives the relative strength of the $^3S_1$ versus $^3P_0$ mechanisms, can be varied. The other free parameter, the overall strength off V_{ann} is of no consequence for A_N while for dσ/dΩ it is chosen so as to obtain the correct experimental total cross section. The values for α and β are kept fixed. With λ = -0.5 one is able to obtain an overall negative A_N at p_{LAB} = 497 MeV/c (T_{LAB} = 123 MeV), although the forward minimum is not deep enough. The relative poorness of the prediction is

however still better than the predictions at similar energies by the nucleon-exchange models [5,7,8] as is shown in fig.2. Both N-exchange models give a forward positive value of A_N at low energy, as long as the final $\pi\pi$ interaction is ignored. On the other hand, when $\pi\pi$ FSI is included in the N-exchange model [11], the forward A_N turns negative but the backward A_N becomes positive instead. As mentioned above, experimental data for A_N at these energies are negative at all angles. By itself, it is amazing that this simple quark model is able to predict observables that show any resemblance to experiment.

The number of partial waves needed at this momentum p_{LAB} is rather small. In fig.3 it is shown that already J<4 and J<5 cannot be distinguished. This means that at this energy J<5 is certainly sufficient.

The prediction at higher momentum, p_{LAB} = 679 MeV/c (T_{LAB} = 210 MeV) with no change in the parameters λ, α, β, and the overall strength of V_{ann}, is given in fig.4. Again the forward negative dip in A_N is not deep enough. It is however encouraging that with the same value λ = -0.5 the quark model now predicts a positive value for A_N near $\cos\theta = 0$, while the negative forward and backward values for A_N remain. The experimental data at this energy[4,5] show the same trend of a double dip with a positive hump at 90 degrees. It is still sufficient at this energy, to include only amplitudes with J<5. For easy comparison also the nucleon-exchange predictions of refs.[5,7,8] are shown in fig.4.

One particularly observes, in this low energy range, the general failure of all models to predict the large forward dip in A_N, simultaneously with a large negative value in the backward direction.

5. Discussion of the results

We note that predictions at low energy for $d\sigma/d\Omega$ and A_N in the simple quark model are better or of the same quality as for N-exchange models. Of course we have a free parameter λ to play with that gives the relative strength of the $^3S_1$ and $^3P_0$ mechanisms, as well as an overall factor that sets the scale of the cross section.

At this point one can ask the question, why can one not do better? To answer that question, it is helpful to explicitly check the angular behaviour of the terms in the sum over J in the amplitudes F_{++} and F_{+-} of eqs.(12,13) which are

$$
\begin{aligned}
F_{++}(\theta) \;\sim\; & -\sqrt{1/2}\;\; {}^3P_0 \;+\; \sqrt{3/2}\;\; (\,{}^3S_1 \;-\; \sqrt{2}\;\; {}^3D_1)\,\cos\theta \\
& +\; \sqrt{5/2}\,(\,\sqrt{2}\,{}^3P_2 \;-\; \sqrt{3}\,{}^3F_2)\,(3\,\cos^2\theta - 1)/2 + \ldots\;,
\end{aligned} \tag{17}
$$

and

$$
\begin{aligned}
F_{+-}(\theta) \;\sim\; & -\sqrt{3/2}\;\; (\,{}^3S_1 \;+\; \sqrt{1/2}\;\; {}^3D_1)\,\sin\theta \\
& -\; \sqrt{5/2}\;\; (\sqrt{1/2}\;\; {}^3P_2 \;+\; \sqrt{1/3}\;\; {}^3F_2)\,(3\,\sin\theta\cos\theta) \;+\; \ldots .
\end{aligned} \tag{18}
$$

From inspection it is clear that for only J=0,1 the asymmetry A_N has the form

$$
A_N \;\sim\; \sin(\theta)\;(a_0 \;+\; a_1\,\cos\,(\theta)). \tag{19}
$$

At low energies, the expression of eq.(19) seems to be the basic form of the predicted A_N in both N-exchange models, and to some extend the same holds for the quark model (see fig.1b). To get an angular behaviour of A_N that is of the pronounced double-dip structure as in experiment, one needs at least a rather substantial J=2 (e.g. ${}^3P_2 -{}^3 F_2$) contribution.

This can be clearly demonstrated by a simple toy-model with only ${}^3P_0$, ${}^3S_1$, and ${}^3F_2$ $\bar{p}p$ amplitudes (replacing ${}^3F_2$ by only ${}^3P_2$ does not work as well due to their opposite signs in F_{++} (θ) in eq.(17)). In fig.5 we show the result of such a toy-model where ${}^3P_0$, ${}^3S_1$, and ${}^3F_2$ have relative strength 1 : 0.8 : 0.5 respectively. The phases of ${}^3P_0$ and ${}^3F_2$ are the same, ${}^3S_1$ has a phase difference of $3\pi/2$ with the other two amplitudes. Here the double-dip structure of the asymmetry A_N, shown in fig.5, is due to a strong J=2 presence.

Going back to the quark model, we observe that indeed there the J=2 amplitude is relatively small compared to the J=0,1 amplitudes, and that is the main reason why A_N has not the pronounced double-dip structure as seen in experiment.

6. Conclusions

One shortcoming of both quarkmodel and N-exchange models may be that the transition potential $V_{\rm ann}$ ($\vec{r}\,'$,$\vec{r}$,$\vec{s}$) is of too short a range to make the J=2 amplitudes play a significant role at low energies. In the simple quark model the range of $V_{\rm ann}$ is determined by the coefficients A,B, and C in the exponent. Their values are set by the nucleon size parameter α and pion size parameter β. In order to increase the range of the potential (and enhance the role of the J=2 amplitude) one can decrease the values of α and β.

As an example we show in fig.6 the result at $p_{\rm LAB}$ = 497 MeV/c for α = 2.8 fm^{-2} and lowering β to 1.73 fm^{-2} (dot-dashed curve). This would correspond with a pion radius of 0.66 fm. The previous value was r_π = 0.48 fm. Indeed one sees that the double-dip structure of A_N is enhanced. At the same time the differential cross section dσ/dΩ is somewhat more forward peaked, in better agreement with the experimental shape. In the case of dσ/dΩ the overall strength was adjusted to give the same value at cos θ = -0.5. The corresponding values for A, B, and C are A = -1.79 fm^{-2}, B = - 3.87 fm^{-2}, and C = - 1.16 fm^{-2}. Similar effects in the asymmetry A_N can be obtained by keeping β fixed at β = 3.23 and reducing α. The dashed curve in fig.6 shows the result for α = 1.24 fm^{-2} (r_p = 0.90 fm). In that case the values for A, B, and C become A = - 0.81 fm^{-2}, B = - 4.52 fm^{-2}, and C = - 0.57 fm^{-2}. For the above two cases the value of λ is - 0.45. In both cases the role of the J=2 amplitudes is enhanced, when compared with the model with the conventional values for α and β (solid curve).

Overall, it seems that experiment requires at this energy a significant J=2 amplitude. One way to accomplish this, is to increase the range of the transition potential as shown above. Of course one can also search for the reason of the discrepancy between theory and experiment, in the model dependence of the initial and final state interaction. Work on this is forthcoming.

Finally, because the reaction $\bar{p}p \rightarrow \pi^\circ\pi^\circ$ has only isospin I = 0 amplitudes and therefore only even J contributions, the measurement of A_N for that process, if ever performed, may shed additional light on the microscopic process of $\bar{p}p$ annihilation into two mesons.

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FIGURE CAPTIONS

FIG. 1. Differential cross section and analyzing power at p_{LAB} = 497 MeV/c. Parameter values are λ = - 0.5, α = 2.80 fm^{-2}, and β = 3.23 fm^{-2}. Data for dσ/dΩ are from ref.[3], and A_N data are from ref.[5].

FIG. 2. Analyzing power at p_{LAB} = 497 MeV/c. The solid curve is the quark model. The nucleon exchange model is represented by the dashed line ref.[5,7], and the dotdash line from ref.(8). All models are without $\pi\pi$ FSI. Data are from ref.[5].

FIG. 3. Effect of various partial waves at p_{LAB} = 497 MeV/c. The solid curve is the prediction for J<5, and cannot be distinguished from J<4. the dashed curve is J<3, and the dotdash curve is J<2.

FIG. 4. Differential cross section and analyzing power at p_{LAB} = 679 MeV/c. The solid curve is the quark model, with the same parameters as in fig.1. The dashed curve is ref.[5,7], and the dotdash curve is ref.[8]. Data for dσ/dΩ are from ref.[3], and A_N data are from ref.[5].

FIG. 5. Result from simple toy-model with $^3P_0$, $^3S_1$, and $^3F_2$ amplitudes. For details see text.

FIG. 6. Results at p_{LAB} = 497 MeV/c for α = 2.8 fm^{-2} and β = 1.73 fm^{-2} (dotdash curve). Dashed curve is for α = 1.24 fm^{-2} and β = 3.23 fm^{-2}. In these first two curves λ = -0.45. The solid curve serves as reference to the earlier values α = 2.8 fm^{-2} and β = 3.23 fm^{-2}, where λ = -0.50. Data are from refs.[3,5].